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# Soil Mechanics I

## 4 – Compressibility

1. Isotropic compression
2. Nonlinearity
3. Overconsolidation; OC vs. creep
4. One-dimensional compression - parameters
5. Settlement (one-dimensional compressibility)
6. At-rest earth coefficient;  $E_{\text{oed}}$  vs  $E'$  for elastic material

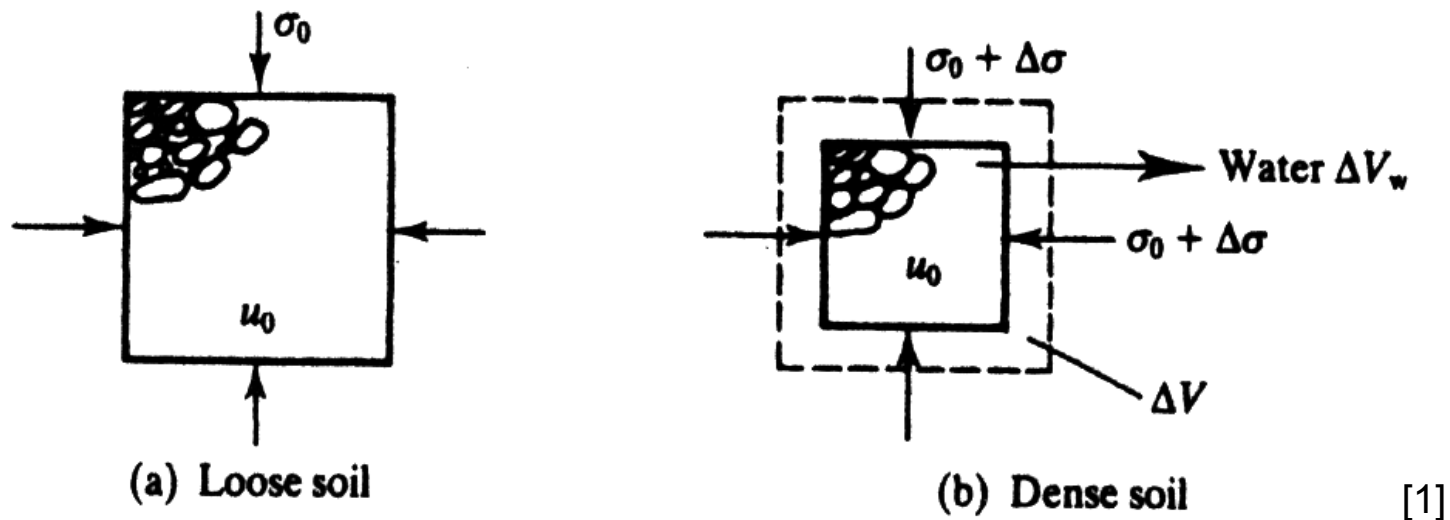
# Isotropic Compression

- Loading
- isotropic  $\Delta\sigma_1' = \Delta\sigma_2' = \Delta\sigma_3' = \Delta\sigma' = \Delta p'$ ;  $q = \text{const.}$
  - anisotropic  $\Delta\sigma_1'$ ;  $\Delta\sigma_2'$ ;  $\Delta\sigma_3' \rightarrow \Delta p'$

$\Delta p' \neq 0 \rightarrow$  structural changes

(changes in position of soil grains) = compression

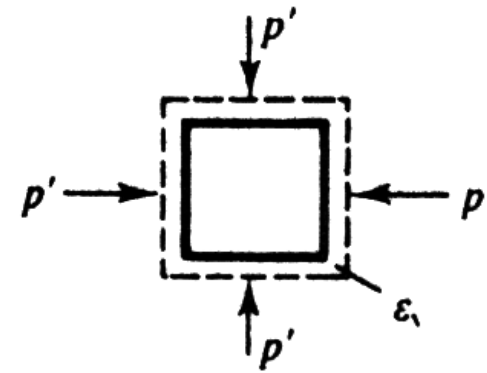
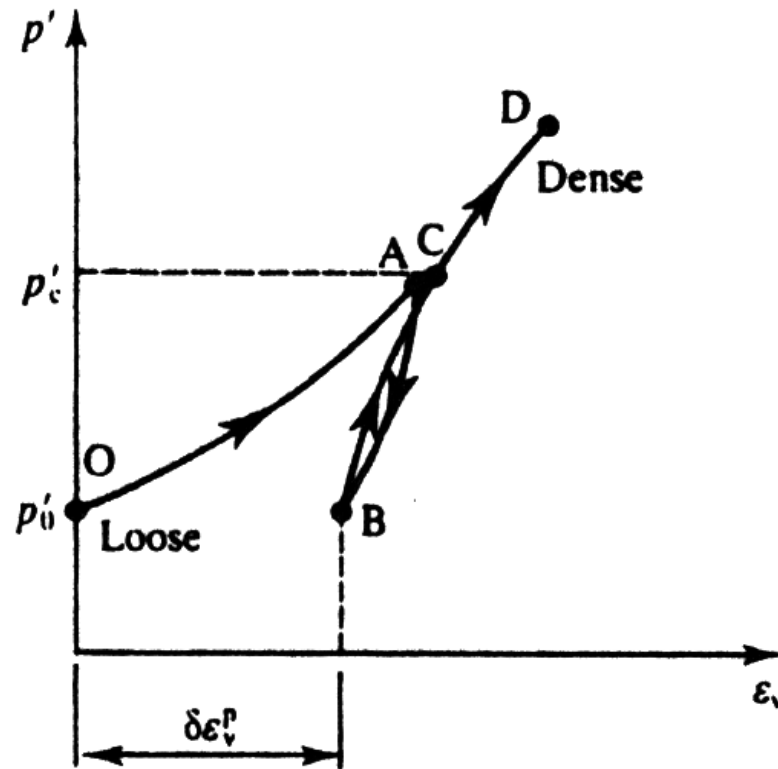
$\Delta p' > 0 \rightarrow$  loose soil  $\rightarrow$  dense soil = change in porosity shown by (a)  $\rightarrow$  (b):



# Isotropic Compression

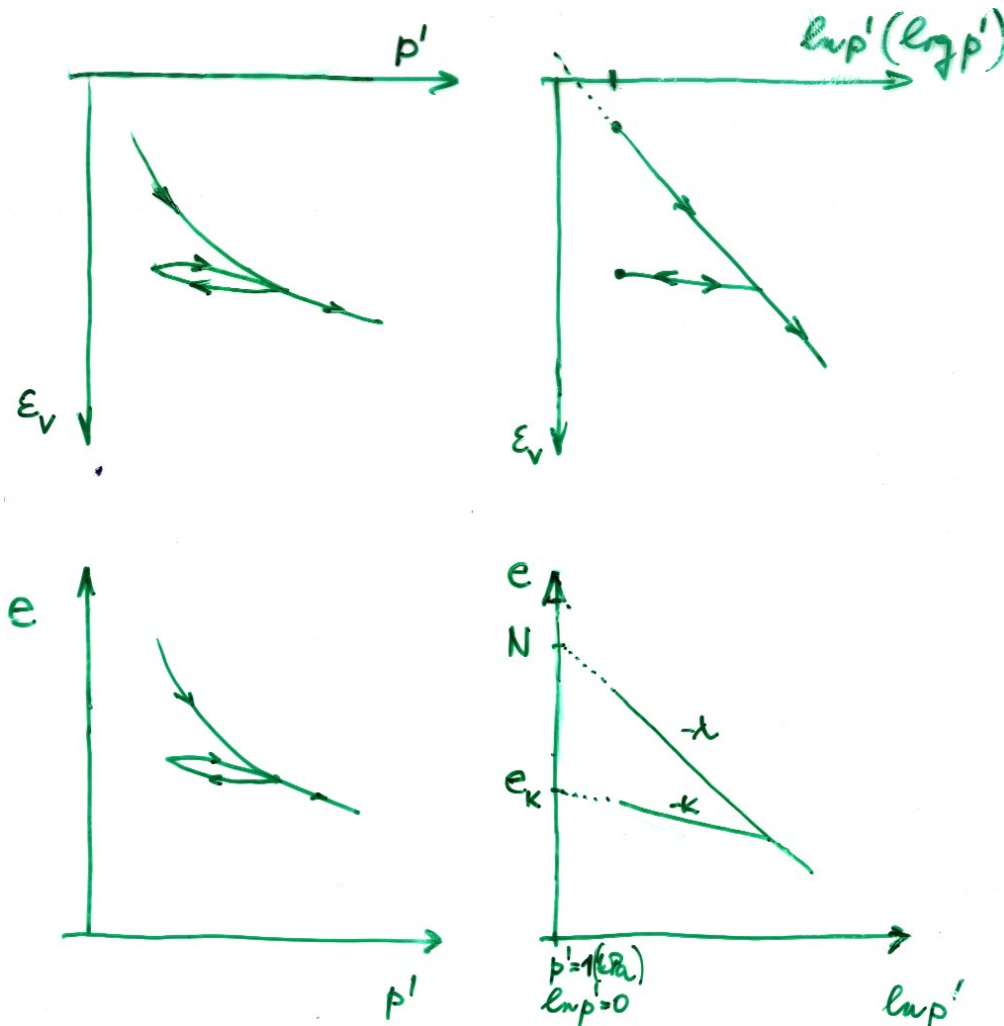
For isotropic compression (i.e. for  $q=\text{const.}$ ):

Bulk Modulus  $K = dp' / d\varepsilon_v \neq \text{const.}$



[1]

# Isotropic Compression



## Isotropic compression

(semi)logarithmic plot in working stress range is **linear**

loading

Normal Compression Line

**NCL:**

$$e = N - \lambda \ln p'$$

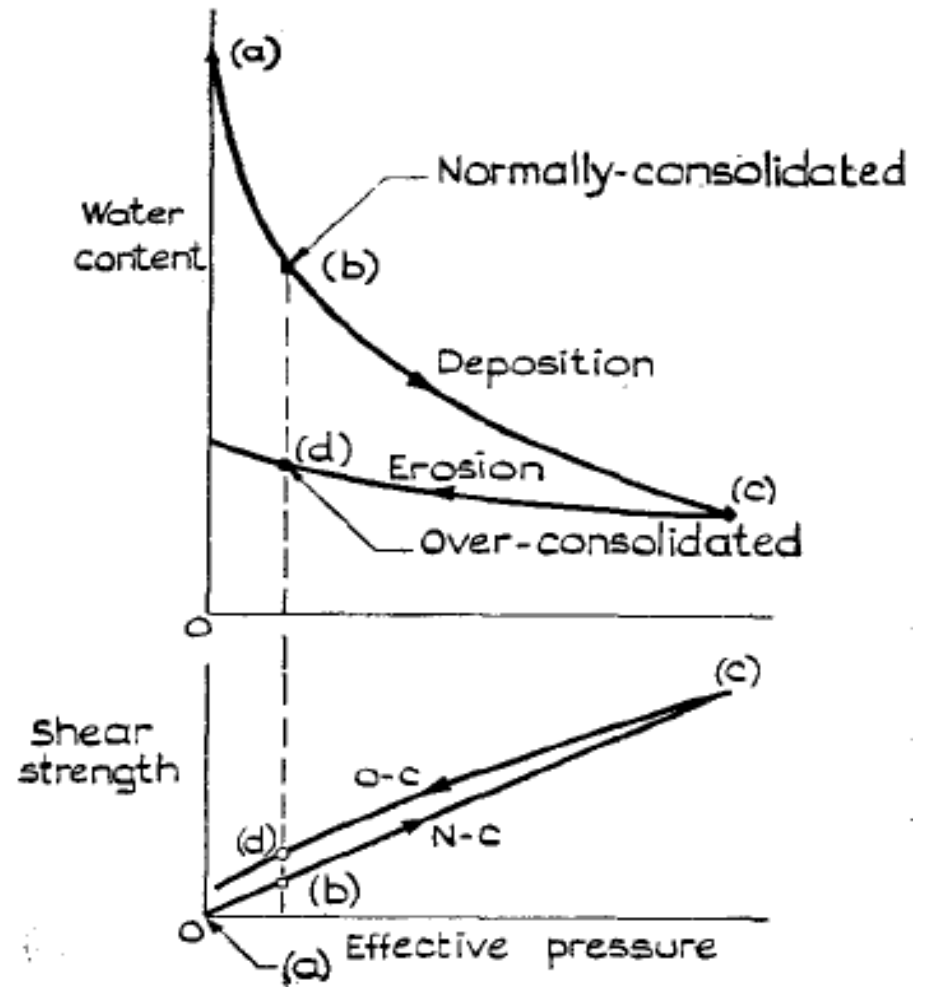
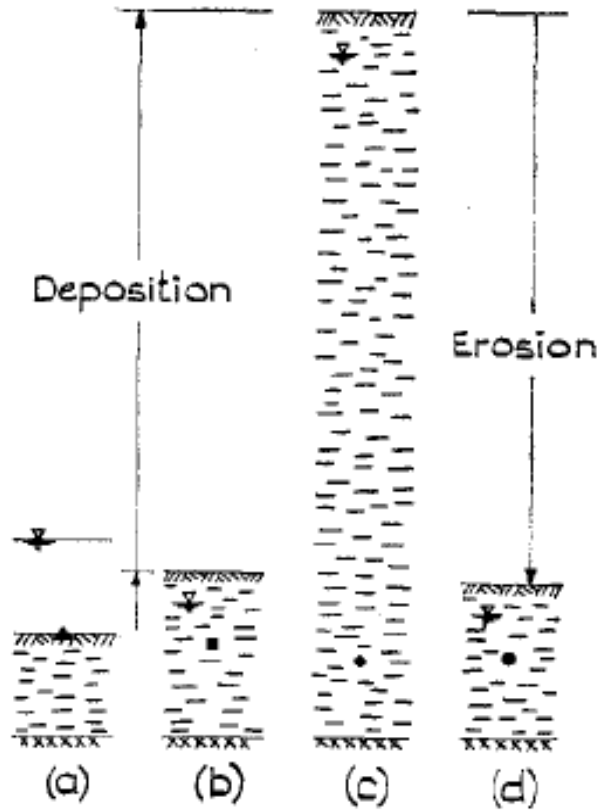
unloading – reloading

$$e = e_k - \kappa \ln p'$$

Parameters for isotropic compression:  $\lambda$ ;  $\kappa$ ;  $N$

They are **constants**

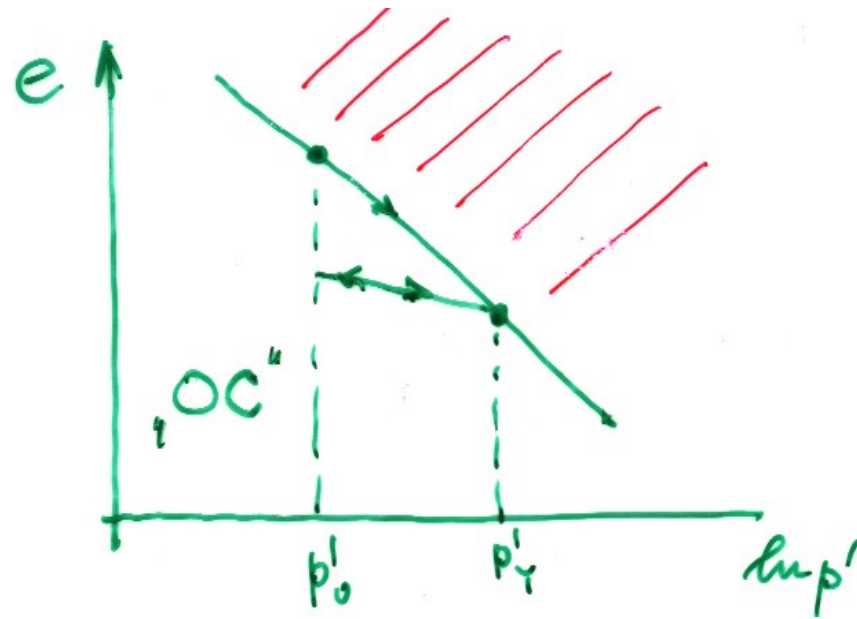
# Overconsolidation



Skempton (1964)

# Overconsolidation

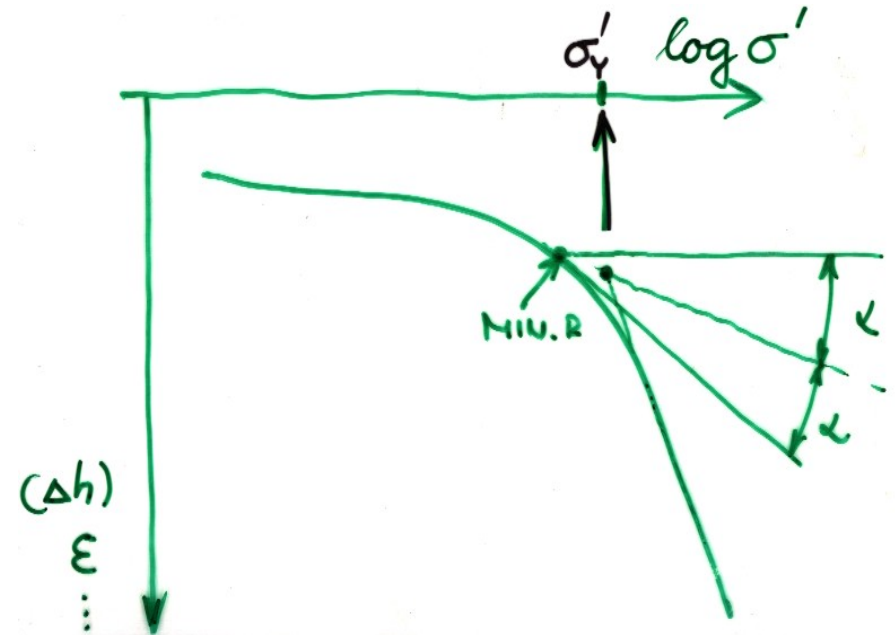
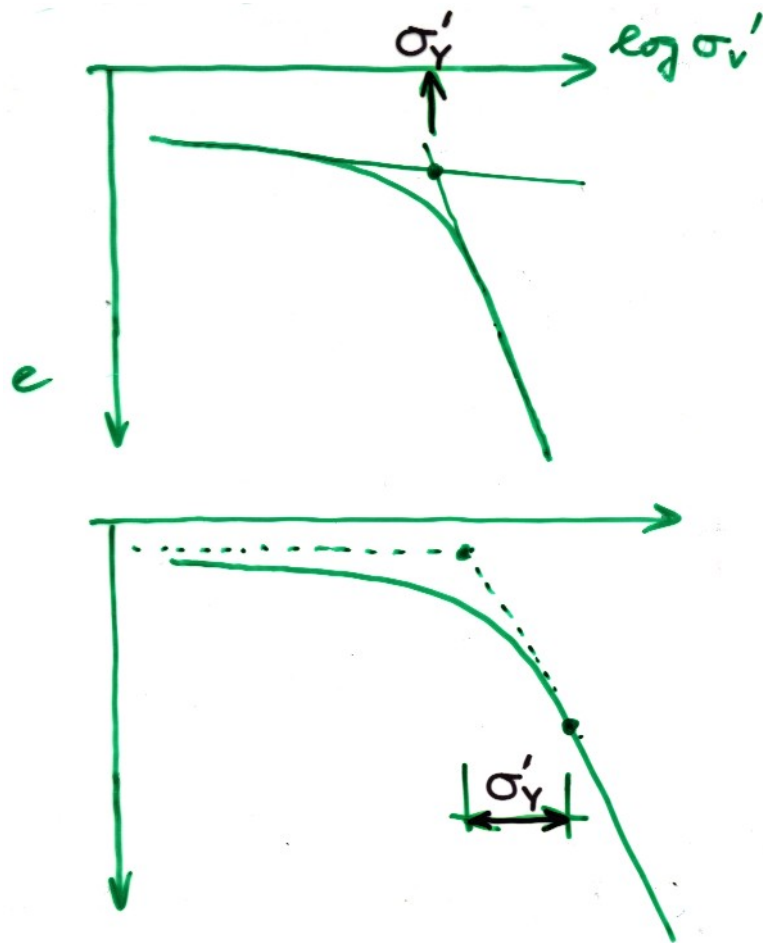
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# Overconsolidation

„overconsolidation stress“ = yield stress  $\sigma'_Y$

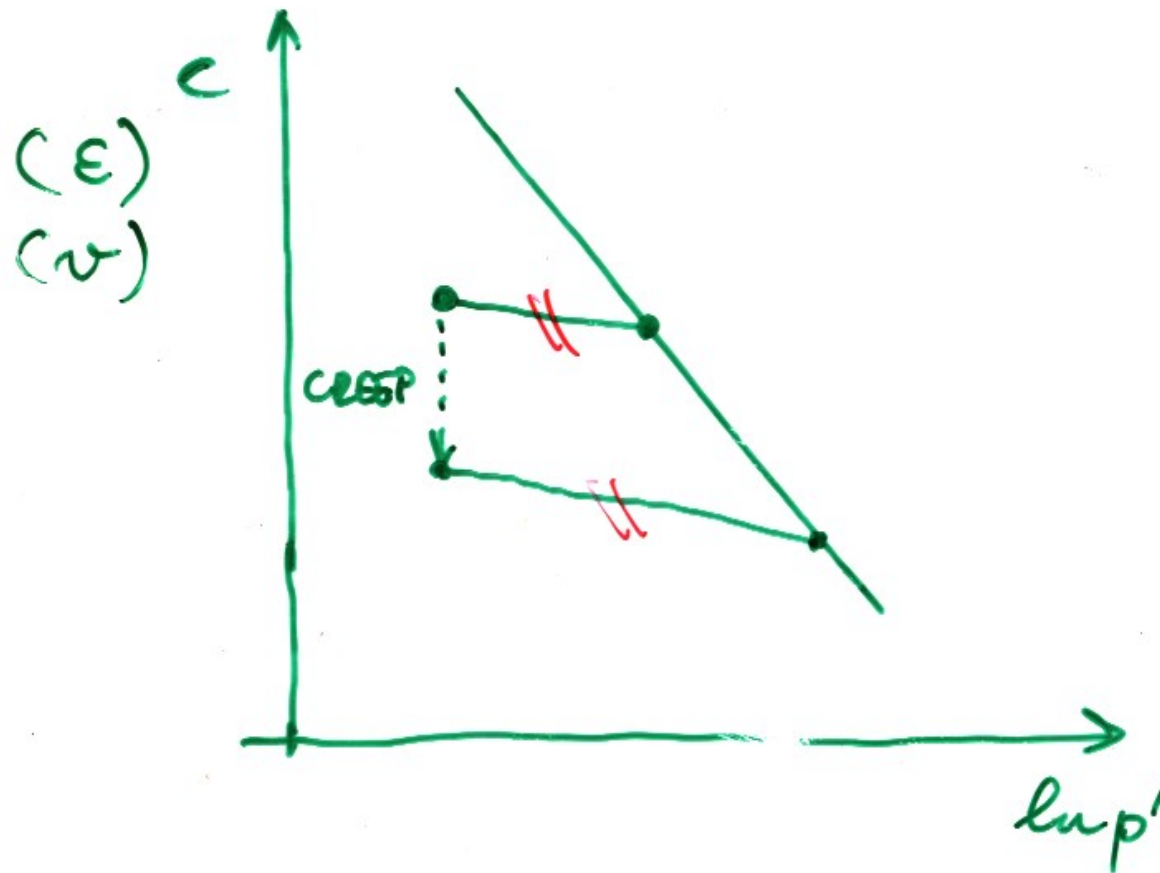
determination (from oedometer data)



# Overconsolidation

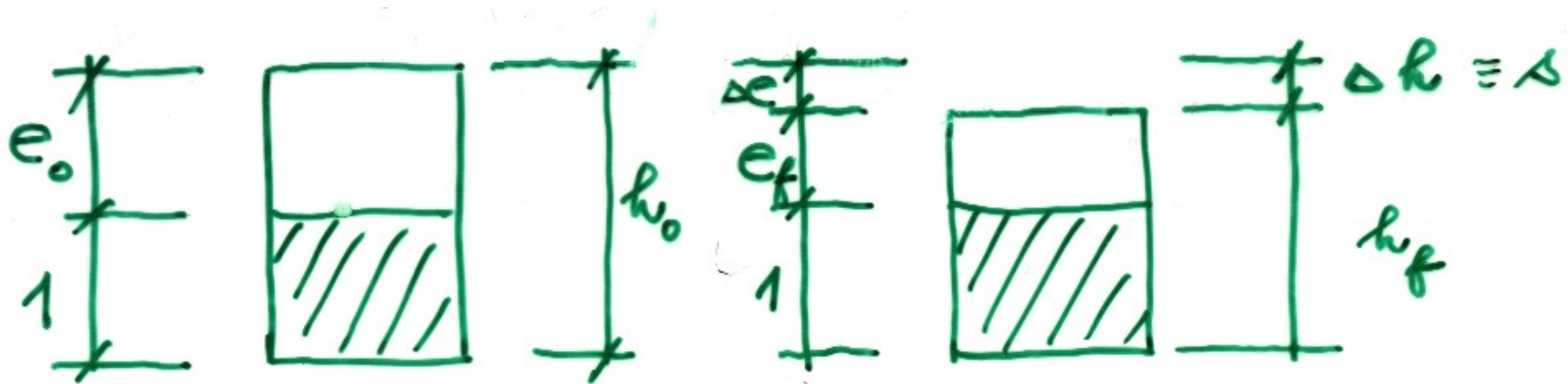
„Overconsolidation Stress“ vs Creep

→ the meaning of „overconsolidation stress“ - ???





## One-dimensional compression (oedometer)



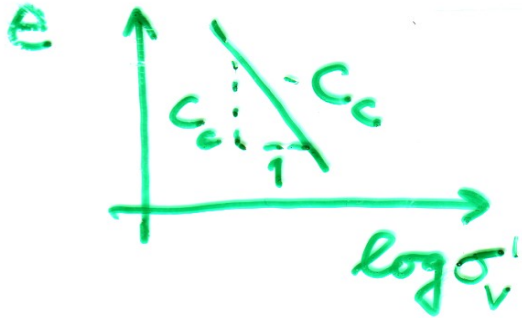
$$\varepsilon_v = -\Delta h / h_0 = -\Delta e / (1+e_0) \equiv s / h_0$$

s – settlement

Settlement calculation:

$$s = -\Delta e / (1+e_0) h_0 = \varepsilon_v h_0$$

# One Dimensional Compression



semi-logarithmic plot of stress vs  $e$ :

$$\text{compressibility index } C_c = - \Delta e / \Delta \log \sigma_v'$$

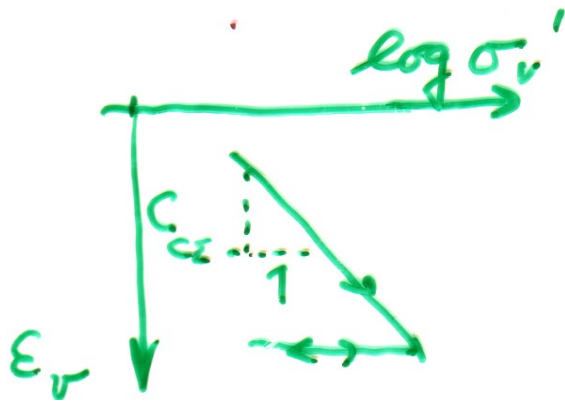
unloading:  $C_{cr}$  (or  $C_s \leftarrow$  'swelling')

$C_c$  is a real parameter – constant (for working stress range)

NB:

In determining  $C_c$ , NCL must be measured (not the compressibility at OC...)

# One Dimensional Compression



semi-logarithmic plot of stress vs  $\varepsilon$ :

modified compressibility index  $C_{ce} = \Delta\varepsilon / \Delta\log\sigma'_v$

unloading  $C_{re}$  (or  $C_{se}$  ← 'swelling')

$$\varepsilon_v = -\Delta e / (1+e_0) \rightarrow C_{ce} = C_c / (1+e_0)$$

# One Dimensional Compression

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arithmetic plot of stress vs  $\epsilon$ :

oedometer modulus  $E_{\text{oed}} = \Delta\sigma_v' / \Delta\epsilon$

coefficient of volume change  $m_v = 1 / E_{\text{oed}}$

Not a real parameter – not a constant – depends on stress level

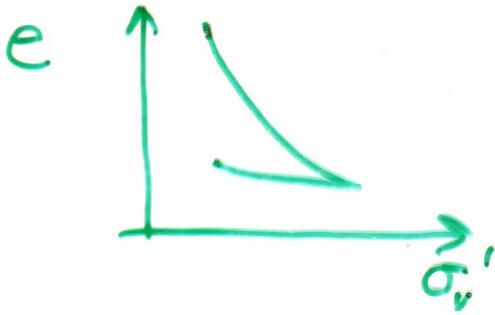
# One Dimensional Compression

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arithmetic plot of stress vs e:

$$\text{coefficient of compressibility } a_v = - \Delta e / \Delta \sigma_v'$$

Not a real parameter – not a constant – depends on stress level



## Settlement computation

$$s = - \Delta e / (1+e_0) h_0 = \varepsilon_v h_0$$

summing over layers:  $s = \sum s_i$

using different „parameters“:

$$s = C_c h_0 / (1+e_0) \log ((\sigma_{v0}' + \Delta\sigma_v') / \sigma_{v0}')$$

$$s = C_{c\varepsilon} h_0 \log ((\sigma_{v0}' + \Delta\sigma_v') / \sigma_{v0}')$$

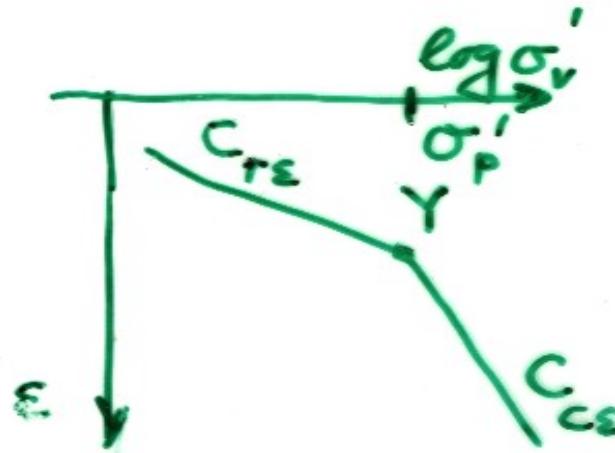
$$s = m_v h_0 \Delta\sigma_v' \quad (m_v \text{ not a constant})$$

$$s = h_0 \Delta\sigma_v' / E_{oed} \quad (E_{oed} \text{ not a constant})$$

$\Delta\sigma_v'$  change in effective stress

## Settlement computation – OC soil

for example with  $C_{c\varepsilon}$



$$s = C_{r\varepsilon} h_0 \log (\sigma_p' / \sigma_{v0}') + C_{c\varepsilon} h_0 \log ((\sigma_{v0}' + \Delta\sigma_v') / \sigma_p')$$

One-dimensional compression – „at rest“ coefficient  $K_0$

$$\varepsilon_h \equiv 0$$

$$\varepsilon_h = 1 / E' (\sigma_h' - \nu' (\sigma_h' + \sigma_v')) = 0$$

$$\sigma_h' - \nu' \sigma_h' - \nu' \sigma_v' = 0$$

$$\sigma_h' / \sigma_v' = \nu' / (1 - \nu') = K_0 = \text{at rest coefficient}$$

**NB:**

- 1 The above definition of  $K_0$  is valid for elastic material only (Hooke's law used);

With soils:  $\nu'$  not a constant

- 2 Effective stress ratio  $K_0$  constant for NC soil only (while NC soil is not elastic at all)



## Elastic relationship between Oedometer and Young's Moduli

Oedometer ( $\Delta\sigma_v$ ;  $\sigma_h' = \sigma_v' K_0$ ;  $\varepsilon_h = 0$ ):

$$\varepsilon_v = \sigma_v' / E_{\text{oed}}'$$

Hooke's Law for a specimen in the oedometer:

$$\varepsilon_v = 1 / E' (\sigma_v' - \nu' (\sigma_h' + \sigma_h')) = \sigma_v' / E' (1 - \nu' (2\nu' / (1 - \nu')))$$

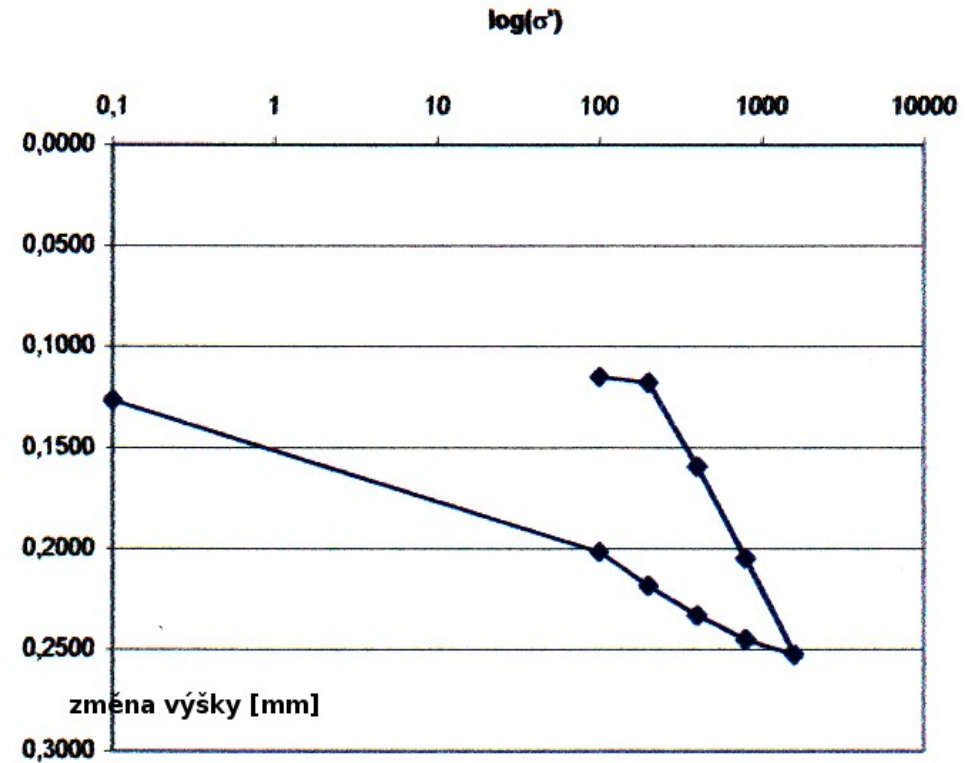
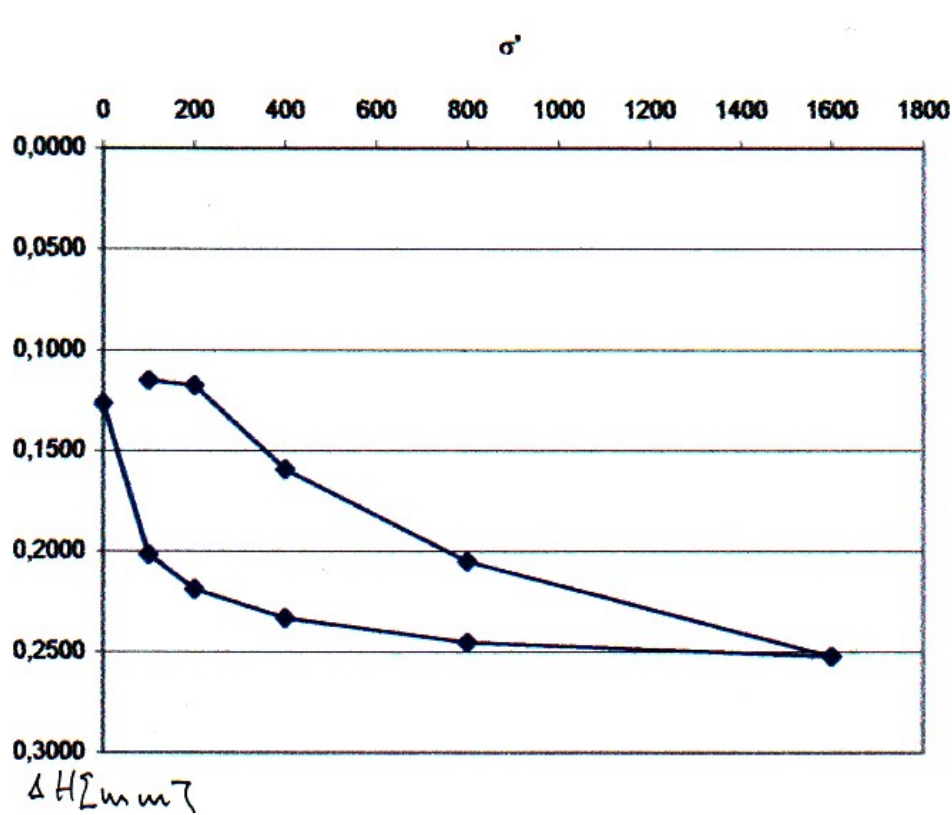
$$\varepsilon_v = \sigma_v' / E' (1 - 2\nu'^2 / (1 - \nu'))$$

$$\rightarrow 1 / E_{\text{oed}}' = 1 / E' (1 - 2\nu'^2 / (1 - \nu'))$$

$$\rightarrow E_{\text{oed}}' = E' / (1 - 2\nu'^2 / (1 - \nu')) \text{ for an elastic material only}$$

# One Dimensional Compression – Lab Class No 7

## Kaolin Clay - Lab class No 7



## Typical values of „parameters“

$$C_c; C_{cr} \approx C_c / 5$$

clay	$C_c = 0,2 - 0,5$
silt, silty clay	$C_c = 0,15 - 0,3$
sensitive clay	$C_c > 1; C_c = 7-10$
peat	$C_c = 10 - 15$

$$E_{oed}$$

fine-grained soil	$E_{oed} = 1 - 30 \text{ MPa}$
kaolin – Lab Class	$E_{oed} = 3 - 8 \text{ MPa}$
sand	$E_{oed} = 5 - 100 \text{ MPa}$
gravel	$E_{oed} = 20 - 500 \text{ MPa}$

Oedometer modulus is stress dependent – not a parameter

<http://labmz1.natur.cuni.cz/~bhc/s/sm1/>

Atkinson, J.H. (2007) The mechanics of soils and foundations. 2<sup>nd</sup> ed. Taylor & Francis.

## Further reading:

Wood, D.M. (1990) Soil behaviour and critical state soil mechanics. Cambridge Univ.Press.

Mitchell, J.K. and Soga, K (2005) Fundamentals of soil behaviour. J Wiley.

Atkinson, J.H: and Bransby, P.L. (1978) The mechanics of soils. McGraw-Hill, ISBN 0-07-084077-2.

Bolton, M. (1979) A guide to soil mechanics. Macmillan Press, ISBN 0-33318931-0.

Craig, R.F. (2004) Soil mechanics. Spon Press.

Holtz, R.D. and Kovacs, E.D. (1981) An introduction to geotechnical engineering, Prentice-Hall, ISBN 0-13-484394-0

Feda, J. (1982) Mechanics of particulate materials, Academia-Elsevier.)

- [1] Atkinson, J.H. (2007) The mechanics of soils and foundations. 2<sup>nd</sup> ed. Taylor & Francis.